SCEGGS Darlinghurst

2005
Higher School Certificate
Trial Examination

Mathematics

This is a TRIAL PAPER only and does not necessarily reflect the content or format of the Higher School Certificate Examination for this subject.

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Diagrams should be drawn in pencil
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 120

- Attempt Questions 1–10
- All questions are of equal value

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Attempt Questions 1–10
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Find the value of $\dfrac{1-0.46^2}{1+0.46^2}$ correct to 3 significant figures. **1**

(b) Write $\dfrac{1}{\sqrt{6}-2}$ with a rational denominator. **2**

(c) If α and β are the roots of $2x^2-6x+3=0$, find the values of:

(i) $\alpha+\beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2}$ **2**

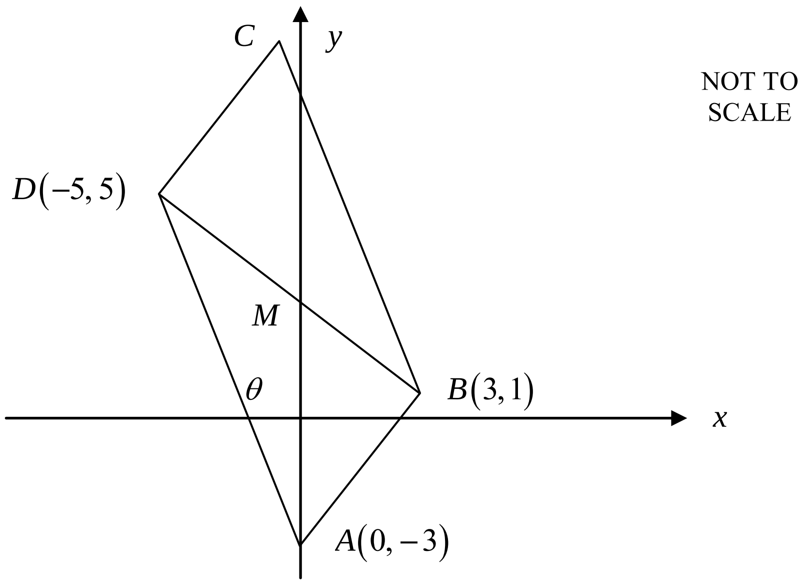
(d) Graph the solution of $|2x+1|\leq 7$ on a number line. **2**

(e) Solve $4^x+3\left(2^x\right)-28=0$ for x. **3**

Marks

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a)



(i) Find the co-ordinates of M, the midpoint of BD. **1**

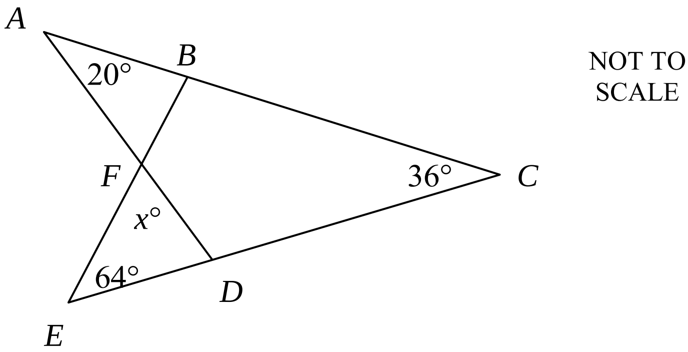
(ii) Find the co-ordinates of C so that $ABCD$ is a parallelogram. **1**

(iii) Show that the line AB has equation $4x - 3y - 9 = 0$. **2**

(iv) Find the perpendicular distance between D and the line AB. **2**

(v) Find the area of parallelogram $ABCD$. **2**

(b) **2**



Copy the diagram above into your answer booklet. Find the value of x giving reasons.

(c) Find the equation of the tangent to the curve $y = \cos 2x$ at $(\pi, 1)$. **2**

Marks

Question 3 (12 marks) Use a SEPARATE writing booklet.

(a) Differentiate:

(i) $y = 7\sqrt{x} - \dfrac{2}{x^3}$ **2**

(ii) $f(x) = \dfrac{2x-1}{3x+1}$ **2**

(b) Beryl does the following solution to solve the equation $2\cos\theta = 1$ for $-\pi \le \theta \le \pi$.

Line 1 $2\cos\theta = 1$

Line 2 $\cos\theta = \dfrac{1}{2}$

Line 3 $\theta = 60°$

(i) Beryl has made at least one mistake in her working. State the line(s) in which the mistake(s) occurred and describe the mistake(s). **2**

(ii) Show the correct solution to the equation. **2**

(c) On a visit to Sydney Harbour, Mary and Frederik sail on their yacht, the Dannebrog, from point A on a course of $077°$ for 20 nautical miles to point B. They then change course to $130°$ and continue sailing for 30 nautical miles to point C.

(i) Draw a neat sketch (at least $\dfrac{1}{3}$ page) depicting this information. **1**

(ii) Show that $\angle ABC = 127°$ **1**

(iii) Given that point A and point C are 45 nautical miles apart find the bearing of point C from their starting point. (Answer correct to the nearest degree.) **2**

Marks

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) Given the function:

$$f(x) = \begin{cases} x-5 & \text{for} \quad x \le 5 \\ (x-5)^2 & \text{for} \quad x > 5 \end{cases}$$

Find:

(i) $f(-2)$ **1**

(ii) $f(a+5)$ when $a > 0$ **1**

(b) Solve for x **3**

$$2\log_e x = \log_e(6-5x)$$

(c) A plant is observed over a period of time. Its initial height is 20 cm. It grows 5 cm during the first week of observation. In each succeeding week the growth, in height, is 80% of the previous week's growth. Assuming this pattern continues, calculate the plant's ultimate height. **3**

(d) A certain parabola has a focus of $(3, 6)$ and a directrix $y = 2$.

(i) Draw a diagram showing this information and the approximate position of the parabola. **1**

(ii) State the co-ordinates of the vertex. **1**

(iii) Write the equation of the parabola in the form $(x-h)^2 = 4a(y-k)$. **1**

(e) "Mrs Brimfield is having twins. She could have 2 boys, 2 girls or a boy and a girl. Therefore, the probability that she has 2 boys is $\frac{1}{3}$." **1**

Is this statement true or false? Give a reason for your answer.

Marks

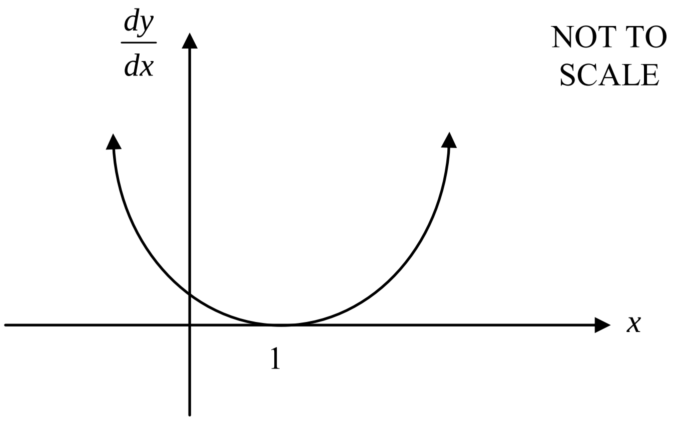
Question 5 (12 marks) Use a SEPARATE writing booklet.

(a) Find the sum of 10 terms of the series **3**

$$\log_m 3 + \log_m 6 + \log_m 12 + \ldots$$

given that $\log_m 3 = 0.48$ and $\log_m 2 = 0.30$

(b) Consider the graph of the derivative $\frac{dy}{dx}$ given below.



(i) Comment on the sign of $\frac{dy}{dx}$ for all x except $x = 1$. **2**

What does this imply about the curve $y = f(x)$ for all x, except $x = 1$?

(ii) What can you conclude about $y = f(x)$ when $x = 1$? **1**

(iii) Sketch a possible graph of $y = f(x)$ **1**

Question 5 continues on page 7

Marks

Question 5 (continued)

(c) A census was taken in 2005 of the population of a coalmining town called Blackrock. The population, P, after t years is given by the exponential equation

$$P = 50000\, e^{-0.08t}$$

(i) What is the initial population of Blackrock in 2005? **1**

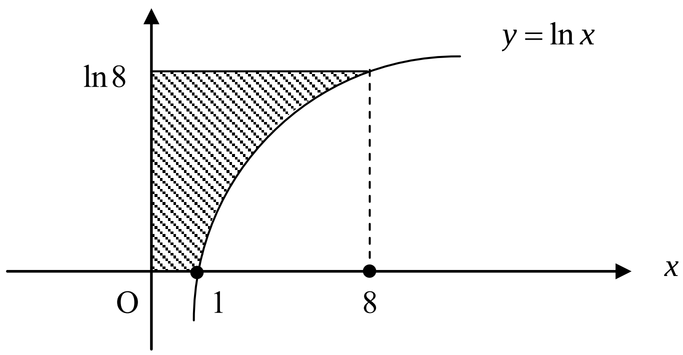
(ii) Find the time in years it will take the initial population to halve. **2**

(iii) At what **rate** is the population changing in 2010? **2**

Marks

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a)



NOT TO SCALE

The diagram shows the area bounded by the graph $y = \ln x$, the co-ordinate axes and the line $y = \ln 8$.

(i) Find the shaded area. **3**

(ii) Hence find the exact value at **1**

$$\int_1^8 \ln x \, dx$$

(b) (i) Solve $(k-1)(k-9) < 0$ **1**

(ii) Find the value of k for which **2**

$$kx^2 + (k+3)x + 4$$

is positive definite.

(iii) Explain why $kx^2 + (k+3)x + 4$ is never negative definite. **1**

Question 6 continues on page 9

Marks

Question 6 (continued)

(c) (i) Explain why $\int_{-\pi}^{\pi} \sin x \, dx = 0$. **2**

(ii) Let m be a positive number. With the aid of a clear diagram, find the number of possible solutions for x, so that $\sin x + mx = 0$ in the domain $-\pi \leq x \leq \pi$. **2**

Marks

Question 7 (12 marks) Use a SEPARATE writing booklet.

(a) The sum of the first three terms of a geometric series is 19 and the sum to infinity is 27.

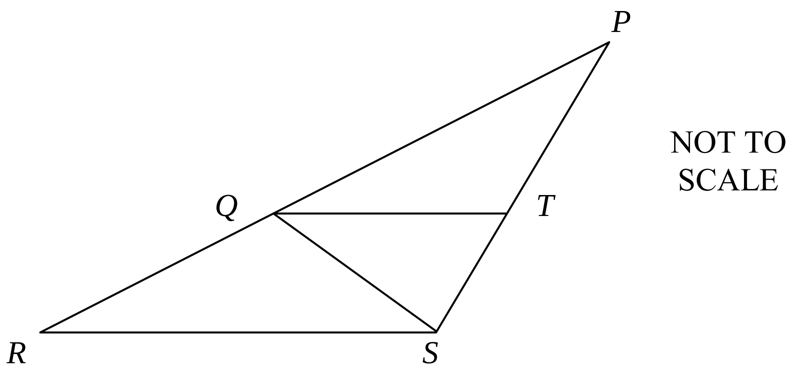
Find:

(i) the value of the common ratio. **2**

(ii) the value of the first term. **1**

(iii) the value of the fifth term. **1**

(b)



In the diagram $QT \parallel RS$ and TQ bisects $\angle PQS$.

Copy the diagram into your answer booklet, showing this information.

(i) Explain why $\angle TQS = \angle QSR$. **1**

(ii) Prove that ΔQRS is isosceles. **2**

(iii) Hence show that $PT : TS = PQ : QS$. **2**

Question 7 continues on page 11

Marks

Question 7 (continued)

(c) In a certain hospitality course all students sit for a theory examination in which 60% of the candidates pass.

Those who pass the theory examination then sit a practical test which is passed by 40% of those who sit the practical test. A student is chosen at random.

Find the probability that:

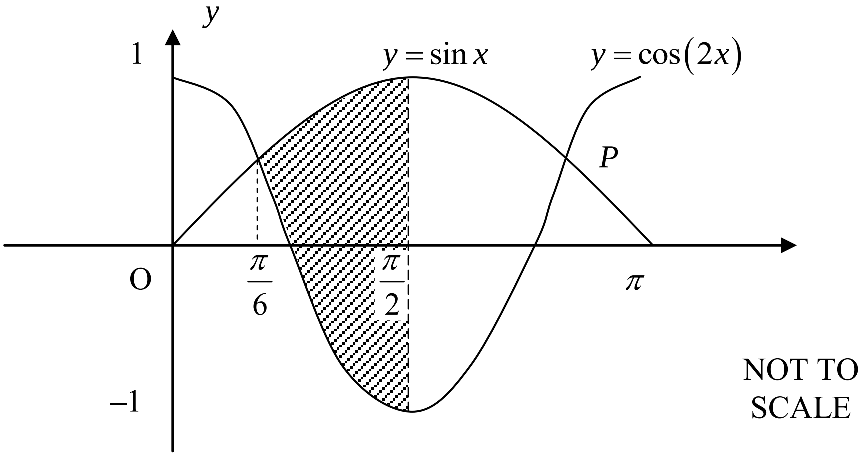
(i) the student passes both examinations. **2**

(ii) the student passes just one of the examinations. **1**

Marks

Question 8 (12 marks) Use a SEPARATE writing booklet.

(a)



The diagram shows parts of the curves $y = \sin x$ and $y = \cos(2x)$.

(i) The curves intersect at $x = \frac{\pi}{6}$. State the co-ordinates of the point P, the other point of intersection in the domain $0 \le x \le \pi$. **2**

(ii) Find the shaded area, leaving your answer in exact form. **4**

(b) Consider the function $f(x) = \frac{1}{2}\left(e^{x} + e^{-x}\right)$

(i) Show that the curve represents an even function. **1**

(ii) Show that the function only has one stationary point and determine its nature. **3**

(iii) Show that the function has no points of inflexion. **1**

(iv) Hence sketch the curve. **1**

Marks

Question 9 (12 marks) Use a SEPARATE writing booklet.

(a) Heather invests \$50000 in an account that earns 8% p.a. interest, compounded annually. She intends to withdraw \$$M$ at the end of each year, immediately after the interest has been paid. She wishes to be able to do this for exactly 20 years, so that the account will then be empty.

(i) Write an expression for the amount of money Heather has in the account immediately after she has made her first withdrawal. **1**

(ii) Write an expression in terms of M for the amount of money in the account, immediately after her 20th withdrawal. **1**

(iii) Calculate the value of M which leaves her account empty after the 20th withdrawal. **2**

(b) (i) Copy and complete the table below, correct to 3 decimal places. **2**

x	2	3	4	5
$\ln 2x$				

(ii) Use the table and the Trapezoidal rule to find an approximation for $\int_2^5 \ln 2x \, dx$ correct to 2 decimal places. **1**

(iii) Sketch a graph of $y = \ln 2x$ and use it to explain whether your approximation in (ii) is an over or under estimate of the exact value of the integral. **2**

(iv) Show that $\frac{d}{dx}\left(x\ln 2x - x\right) = \ln 2x$ **1**

(v) Hence, deduce the exact value of $\int_2^5 \ln 2x \, dx$. **2**

Marks

Question 10 (12 marks) Use a SEPARATE writing booklet.

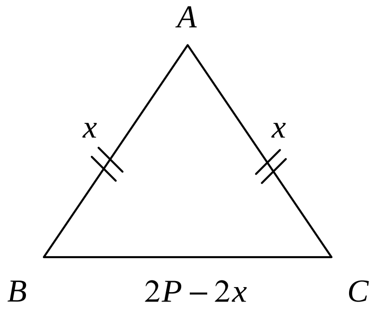
(a) A machine produces Mathomats of which 5% are defective.

(i) What is the probability that a Mathomat is NOT defective? **1**

(ii) A random sample of n items is taken from the machine. **2**
Find the largest value of n that must be sampled so that the probability that none of the Mathomats are defective is at least 0.5.

(b) Find the volume of the solid of revolution formed when the area bounded by the curve $y = \frac{1}{\sqrt{2x+1}}$, $x = 0, x = 1$ and the x axis is rotated about the x axis. **3**

(c) ΔABC is an isosceles triangle of constant perimeter $2P$ and equal sides of length x.



NOT TO SCALE

(i) Show that the area of the triangle, A, can be given by the expression: **2**

$$A = (P - x)\sqrt{2Px - P^2}$$

(ii) Show that $\frac{dA}{dx} = \frac{P(P-x)}{\sqrt{2Px - P^2}} - \sqrt{2Px - P^2}$ **2**

(iii) Hence, show that the maximum area of all isosceles triangles of constant perimeter $2P$ occurs when ΔABC is equilateral. **2**

End of Paper

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STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

2 Unit TRIAL 2005

1 a) $0.650709805 \doteq 0.651$ (1)

b) $\frac{1}{\sqrt{6}-2} \times \frac{\sqrt{6}+2}{\sqrt{6}+2}$ (1) $= \frac{\sqrt{6}+2}{6-4} = \frac{\sqrt{6}+2}{2}$ (1)

Can't cancel 2 and −4.

c) (i) $\alpha + \beta = \frac{6}{2} = 3$ (1)

You must learn formulae

(ii) $\alpha\beta = \frac{3}{2}$ (1)

(iii) $\frac{\beta^2+\alpha^2}{\alpha^2\beta^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$ (1)

Don't guess.

$= \frac{3^2 - 2\times\frac{3}{2}}{\left(\frac{3}{2}\right)^2} = \frac{9-3}{\frac{9}{4}} = 6\times\frac{4}{9}$

(1) $= \frac{8}{3} = 2\frac{2}{3}$

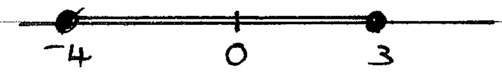
d) $|2x+1| \le 7$

$-7 \le 2x+1 \le 7$

$-8 \le 2x \le 6$

$-4 \le x \le 3$ (1)

Many forgot how to do these questions



1 comm

e) Let $m = 2^x$

$m^2 + 3m - 28 = 0$ (1)

$(m+7)(m-4) = 0$

$m = -7, 4$ (1)

$\therefore 2^x = -7 \quad 2^x = 4$

$\therefore$ no soln $+$, $x = 2$ (1) ← must have both

Reduce to a quadratic equation and you must state $x^2 = -7$ has no solution.

2 a) (i) $M = (-1, 3)$ (1)

(ii) $\frac{x+0}{2} = -1 \qquad \frac{y-3}{2} = 3$

$x = -2 \qquad y - 3 = 6$

$y = 9$

$\therefore C = (-2, 9)$ (1)

(iii) $m = \frac{4}{3}$

$y - 1 = \frac{4}{3}(x-3)$

$3y - 3 = 4x - 12$ comm 2

$4x - 3y - 9 = 0$

(iv) $d = \left| \dfrac{4\times 5 - 3\times 5 - 9}{\sqrt{16+9}} \right|$ (1)

$= \dfrac{44}{5}$ (1)

(v) $AB = \sqrt{(3-0)^2 + (1+3)^2}$ (1)

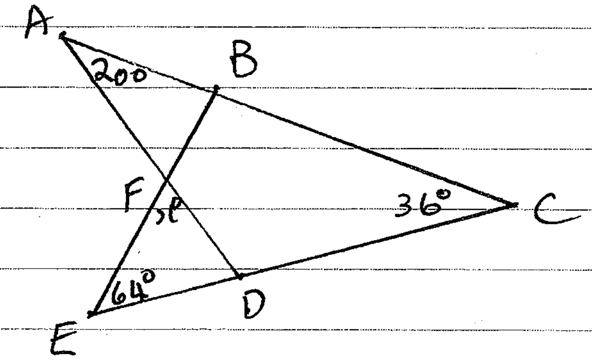
$= \sqrt{9+16} = 5$

$\therefore$ Area $ABCD = \dfrac{44}{5} \times 5$

$= 44\ u^2$ (1)

If using $A = \frac{1}{2}h \times b$ the base is AB, many students used AD.

b)



$\angle ADC = 180° - 20° - 36°$ (1)

$= 124°$ ($\angle$ sum of $\Delta = 180°$)

$x° = 124° - 64° = 60°$ (ext. $\angle$ of Δ = sum int. opp $\angle$'s) (1)

You must give clear and accurate reasons for every step in your working

c) $\dfrac{dy}{dx} = -2\sin 2x$

Don't forget the negative

when $x = \pi$, $m_{tan} = -2\sin 2\pi$

$= 0$ (1)

evaluate $-2\sin 2\pi$ to get 0.

$\therefore y - 1 = 0(x - \pi)$

$\therefore y = 1$ (1)

3 a) (i) $y = 7x^{\frac{1}{2}} - 2x^{-3}$

$\dfrac{dy}{dx} = 7 \cdot \frac{1}{2} x^{-\frac{1}{2}} + 6x^{-4}$ (1) (1)

$= \dfrac{7}{2\sqrt{x}} + \dfrac{6}{x^4}$

Learn and practise index laws.

Calc 4

(ii) $f'(x) = \dfrac{2(3x+1) - 3(2x-1)}{(3x+1)^2}$ (1)

$= \frac{6x+2-6x+3}{(3x+1)^2} = \frac{5}{(3x+1)^2}$ (1)

b) (i) Line 3. θ must be in radians. (1)

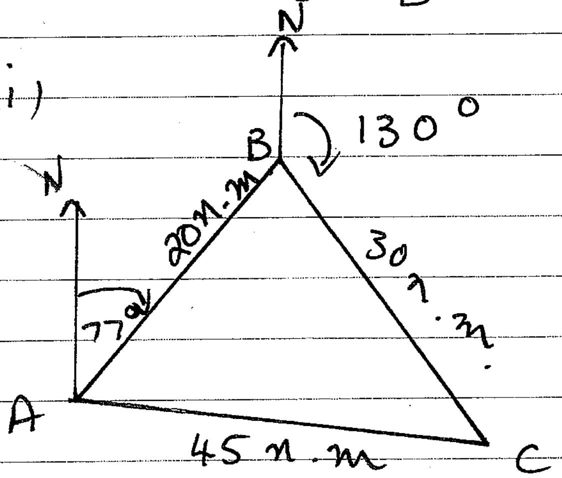
Line 3. There are 2 solutions (1)

Comm 2

Be brief with your explanations

(ii) $\theta = -\frac{\pi}{3}, \frac{\pi}{3}$ (2)

c) (i)



1 Comm

(ii) $\angle ABN = 180° - 77° = 103°$ (coint. ∠'s on ∥ lines add to 180°) (1)

$\therefore \angle ABC = 360° - 130° - 103°$

$= 127°$ (∠'s at pt. add to 360°)

(iii) $\cos \angle BAC = (20^2 + 45^2 - 30^2) \div 2 \times 20 \times 45$ (1)

$\therefore \angle BAC \doteq 32°$

∴ The bearing of C from A is 109° T (1)

Reas 2

You can also use the sine rule to find ∠BAC. Many did not then state the bearing of C from A.

4/ a) (i) $f(-2) = -2-5 = -7$ (1)

well done

(ii) $f(a+5) = (a+5-5)^2 = a^2$ (1)

Simplify in brackets first

b) $\ln x^2 = \ln(6-5x)$ (1)

Know log rules.

$\therefore x^2 = 6-5x$

$x^2 + 5x - 6 = 0$

Reas 3

$(x-1)(x+6)=0$ (1)

$\therefore x = 1, -6$

$\therefore x = -6$ is not a soln.

Always check your solutions.

$\therefore x = 1$ (1)

c) $20 + 5 + 5 \times 0.8 + 5 \times 0.8^2 + \dots +$ (1)

For S_∞ $|r|$ must be < 1.

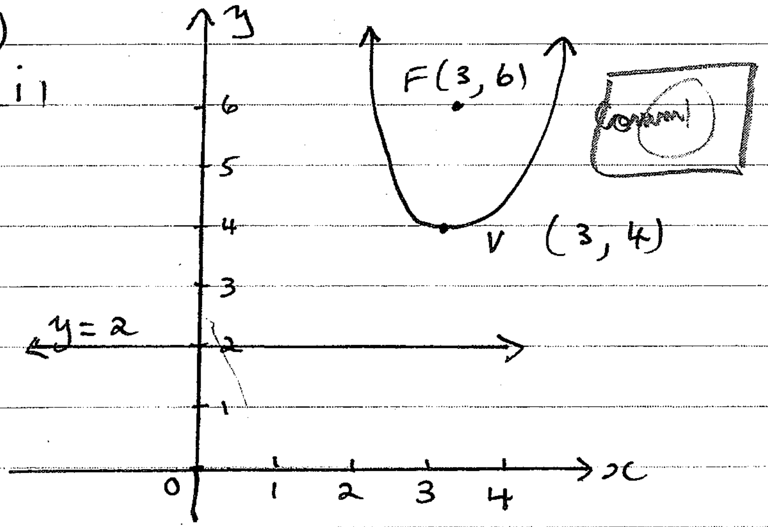
Reas 3

$S_\infty = \frac{5}{1-0.8} = 25$ (1)

$\therefore$ Total height $= 20 + 25 = 45\text{cm}$ (1)

d)

i)



Well done

(ii) $V = (3, 4)$ (1)

Well done

(iii) $(x-3)^2 = 4.2(y-4)$

$(x-3)^2 = 8(y-4)$ (1)

Well done.

e) false as there are 2 chances of having B or G. (1) comm

Probability tree helps.

5 a) $\log_m 3 + \log_m(2 \times 3) + \log_m(2^2 \times 3) + \dots$

$\therefore$ A.P. $a = \log_m 3$, $d = \log_m 2$ (1)

Reas 3

$S_{10} = \frac{10}{2}(2 \times \log_m 3 + 9 \log_m 2)$ (1)

$= 5(2 \times 0.48 + 9 \times 0.3)$

$= 18.3$ (1)

This is an AP not a GP. Use log laws to break down each term, then you will see the difference of $\log_m 2$

b) i) $\frac{dy}{dx}$ is +ve for all x except when $x = 1$ and it equals 0. This implies the curve (1)

Calc 4

2 marks and 2 parts to the question so 2 statements must be made. 1 - comment on sign of $\frac{dy}{dx}$, 2 - what does this imply.

is an increasing fn. except when $x = 1$. (1)

(ii) When $x = 1$, there is a stationary point on $y = f(x)$ as $\frac{dy}{dx} = 0$

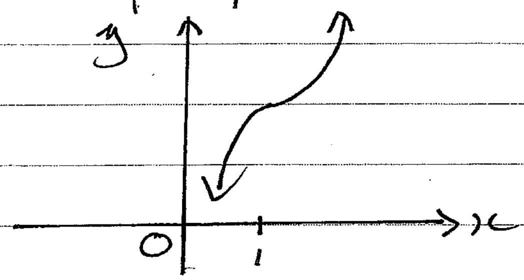
and as $x < 1$ $\frac{dy}{dx} > 0$

and as $x > 1$ $\frac{dy}{dx} > 0$

ie. at $x = 1$ there is a horizontal point of inflexion (1)

You must state that it is a horizontal point of inflextion, not just a stationary point.

(iii)



(1)

you must clearly show the horizontal POI at $x = 1$

c) (i) $t = 0 \therefore P_{initial} = 50000$ (1)

(ii) $2500 = 50000\, e^{-0.8t}$

(1) $\frac{1}{2} = e^{-0.08t}$

$\ln \frac{1}{2} = -0.08t$

$t = \ln \frac{1}{2} \div -0.08$

$\doteqdot 9$ yrs (1)

Be careful when transcribing numbers. A few used 5000 instead of 50 000

(iii) $\frac{dP}{dt} = 50000 \times -0.08\, e^{-0.08t}$ (1)

when $t = 5$, $= -2681.280$

ie. 2681 people/yr. (1)

6. a) (i) $y = \log_e x$

$x = e^y$ (1)

$A = \int_0^{\ln 8} e^y \, dy = [e^y]_0^{\ln 8}$ (1)

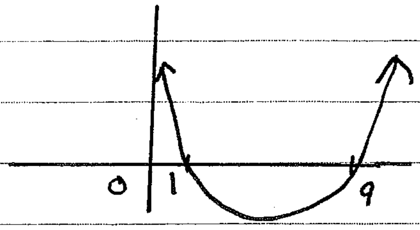
$= e^{\ln 8} - e^0 = 8 - 1 = 7$ (1)

Calc 4

(ii) $\int_1^8 \ln x \, dx = 8 \times \ln 8 - 7$

$= 8 \ln 8 - 7$ (1)

b) (i)



(1)

$\therefore 1 < k < 9$

(ii) +ve definite $a > 0$ & $\Delta < 0$

ie. $k > 0$ (1) and

$(k+3)^2 - 4.k.4 < 0$ (1)

$k^2 - 10k + 9 < 0$

$(k-9)(k-1) < 0$

$\therefore 1 < k < 9$

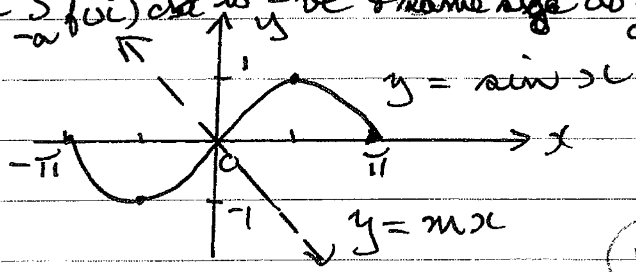
Comm 1 (iii) +ve definite means the value of $kx^2 + (k+3)x + 4$ is always (1) +ve $\therefore$ can't be -ve definite.

Comm 2 c) (i) sine is an odd function

$\therefore \int_{-a}^{a} f(x) = 0$ (1)

and $\int_{-a}^{0} f(x)\,dx$ is -ve & same size as $\int_{0}^{a} f(x)\,dx$ (1)

Reas 2 (ii)



(1)

$\therefore$ only 1 possible soln. (1)

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7. a) $S_3 = \dfrac{a(1-r^3)}{1-r} = 19$ ① (1)

$S_\infty = \dfrac{a}{1-r} = 27$ ② (1)

sub. ② in ① $\quad 27(1-r^3) = 19$

$1 - r^3 = \dfrac{19}{27}$

$1 - \dfrac{19}{27} = r^3$

$\therefore r^3 = \dfrac{8}{27}$

If your calculations get out of hand stop and look for another method of solution it is only worth 1 mark.

$\therefore r = \frac{2}{3}$ (1)

(ii) $\frac{a}{1-\frac{2}{3}} = 27$

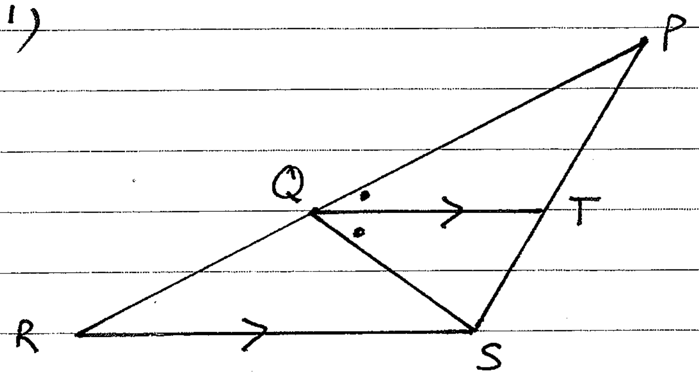
$3a = 27$ (1)

$\therefore a = 9$

(iii) $T_5 = 9 \times \left(\frac{2}{3}\right)^4 = \frac{16}{9}$ (1)

If your $r > 1$ then no CFFPE for parts (ii) and (iii) as can't find S_∞.

b) (i)



Comm 1

(i) $\angle TQS = \angle QSR$ (altn ∠'s = on ∥ lines) (1)

(ii) $\angle QRS = \angle PQT$ (corresp ∠ = on ∥ lines)

$\therefore \angle QRS = \angle QSR$ from above

$\therefore \Delta QRS$ is isosceles.

Again make sure your reasons are clear and accurate. Take care when writing out angles ie. don't mix up $\angle QSR$ & $\angle QRS$.

Reas 2

(iii) $\frac{PQ}{QR} = \frac{PT}{TS}$ (1) (∥ lines cut off = intercepts on transversals)

but $QR = QS$ as ΔQRS is isosceles

$\therefore \frac{PT}{TS} = \frac{PQ}{QS}$ (1)

c)

I		P
0.6 P	0.4	P
	0.6	F
0.4 F		

(1)

(i) $P(PP) = 0.6 \times 0.4 = 0.24$ or 24% (1)

(ii) $P(P\ one) = 0.6 \times 0.6 = 0.36$ or 36% (1)

Very well done! - The people who drew tree diagrams had the greatest success.

Calc 12

8. a) (i) $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ (1)

$\left(\frac{5\pi}{6}, \frac{1}{2}\right)$ (1)

Use symmetry to find x. Many left out '$\frac{1}{2}$'

(ii) $A = \int_{\pi/6}^{\pi/2} \sin x - \cos 2x \, dx$ (1)

$= \left[-\cos x - \frac{1}{2}\sin 2x\right]_{\pi/6}^{\pi/2}$ (1)

$= \left(0 - \frac{1}{2}\times 0\right) - \left(-\frac{\sqrt{3}}{2} - \frac{1}{2}\times\frac{\sqrt{3}}{2}\right)$ (1)

$= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4} = \frac{3\sqrt{3}}{4}$ (1)

Well done

Be careful when integrating

b) (i) $f(-x) = \frac{1}{2}(e^{-x} + e^{x}) = f(x)$

$\therefore$ even fn (1)

Well done.

(ii) $f'(x) = \frac{1}{2}(e^{x} - e^{-x})$

$\frac{1}{2}(e^{x} - e^{-x}) = 0$

$e^{x} = e^{-x}$

$\therefore x = 0$ (1)

$f''(x) = \frac{1}{2}(e^{x} + e^{-x})$ (1)

$f''(0) = \frac{1}{2}(1+1) = 1$

$\therefore$ min at $(0, 1)$ (1)

Know your derivatives.

If solution is not obvious take logs of both sides.

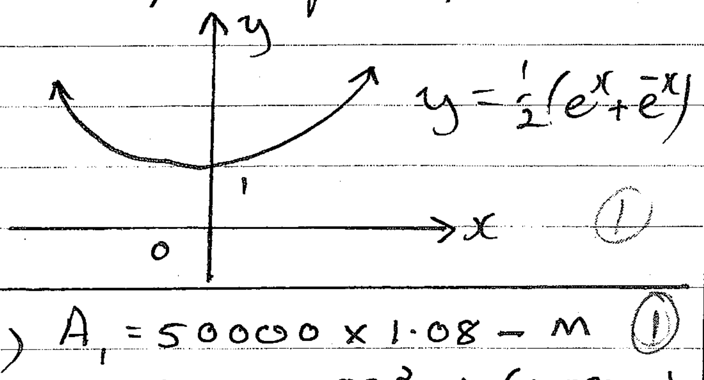
(iii) $f''(x) = \frac{1}{2}(e^{x} + e^{-x}) = 0$

$\therefore e^{x} = -e^{-x}$

no soln. (1)

$\therefore$ no pt. of inflex

(iv)



(1)

Put y intersept on graph

9 a) (i) $A_1 = 50000 \times 1.08 - M$ (1)

(ii) $A_2 = 50000 \times 1.08^2 - M(1.08 + 1)$

$\vdots$ (1)

$A_{20} = 50000 \times 1.08^{20} - M(1.08^{19} + \dots + 1)$

(iii) $M = \dfrac{50000 \times 1.08^{20}}{1.08^{19} + \dots + 1}$ (1)

$= \dfrac{50000 \times 1.08^{20}(1.08 - 1)}{1(1.08^{20} - 1)}$

Many were confused with indices inside brackets

$M = \$5092.60$ (1)

b) (i)

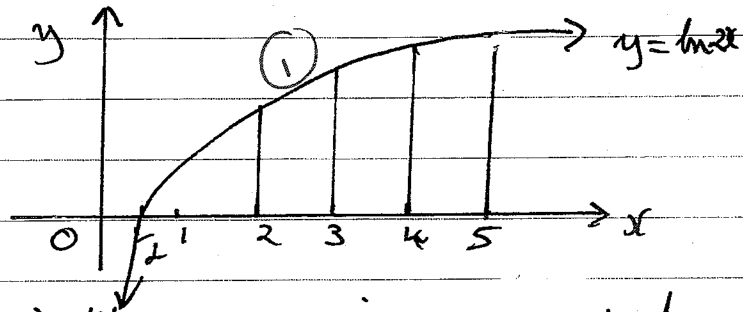
x	$\ln 2x$	w	$w \times \ln 2x$
2	1.386	1	1.386
3	1.792	2	3.584
4	2.079	2	4.159
5	2.303	1	2.303
			11.432

(2)

Read the question. You were asked to complete the table correct to 3 d.p.

(ii) $A = \frac{1}{2} \times 11.432 \doteqdot 5.72$ (1)

(iii)



Comm 2

To get the mark for the graph you needed to show the correct x intercept

Since the curve is concave down, all the trapezia will lie under the curve ∴ the approx will be less than the exact value of the integral (1)

(iv) $\frac{d}{dx}(x \ln 2x - x)$

$= \ln 2x + \frac{2}{2x} \cdot x - 1$ (1)

$= \ln 2x + 1 - 1 = \ln 2x$

Many tried to fudge this answer you should show clearly especially $\frac{2}{2x} \cdot x$

(v) $\int_2^5 \ln 2x \, dx = \left[x \ln 2x - x\right]_2^5$ (1)

Reas 2

$= (5 \ln 10 - 5) - (2 \ln 4 - 2)$

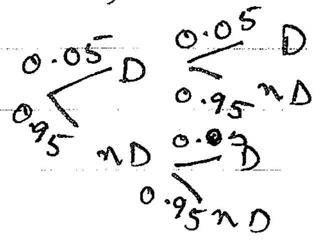
$= \ln 100000 - \ln 16 - 3$

$= \ln \frac{100000}{16} - 3$ (1) $= \ln 6250 - 3$

Careful how you simplify this answer.

10. a) (i) $P(nD) = 95\%$ (1)

(ii)



etc

$(0.95)^n > 0.5$ (1)

$\ln(0.95^n) > \ln 0.5$

$n \ln 0.95 > \ln 0.5$

$n < \dfrac{\ln 0.5}{\ln 0.95}$

$n < 13.51340733$

$\therefore n = 13$ (1)

b) $V = \pi \int_0^1 \frac{1}{2x+1}\,dx$ (1)

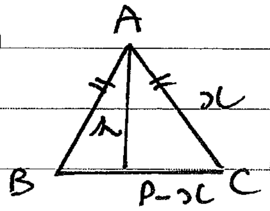
Calc 3

$= \frac{\pi}{2}\left[\ln(2x+1)\right]_0^1$ (1)

$= \frac{\pi}{2}(\ln 3 - \ln 1)$

$= \frac{\pi}{2} \ln 3 \quad u^3$ (1)

c)(i)



Reas 6

$h^2 = x^2 - (P-x)^2$

$= x^2 - (P^2 - 2Px + x^2)$

(1) $= -P^2 + 2Px$

$\therefore h = \sqrt{2Px - P^2}$

$\therefore A = \frac{1}{2}(2P - 2x)\sqrt{2Px - P^2}$ (1)

$= (P-x)\sqrt{2Px - P^2}$

(ii) $\dfrac{dA}{dx} = -1\sqrt{2Px-P^2} + (P-x)\frac{1}{2}2P(2Px-P^2)^{-\frac{1}{2}}$ (1) (1)

$= \dfrac{P(P-x)}{\sqrt{2Px-P^2}} - \sqrt{2Px-P^2}$

(iii) $\sqrt{2Px-P^2} = \dfrac{P(P-x)}{\sqrt{2P-P^2}}$

$2Px - P^2 = P^2 - Px$

$3Px = 2P^2$

$x = \dfrac{2P}{3}$ (1)

$\therefore$ sides are $\frac{2P}{3}, \frac{2P}{3}, 2P - \frac{4P}{3} = \frac{2P}{3}$

$\therefore$ perimeter $= 3 \times \frac{2P}{3} = 2P$

(1)

x	$\frac{P}{3}$	$\frac{2P}{3}$	P
$\frac{dA}{dx}$	+ve	0	−ve

(1)

$\therefore$ max area when $x = \frac{2P}{3}$